2023

MATHEMATICS — GENERAL

Paper : SEC-B-2

(Boolean Algebra)

Full Marks : 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

(Marks : 20)

1. Choose the correct option and justify your answer : (1+1)×10

(a) Let $S = \{1,2,3\}$ and consider the relation $R = \{(1,1), (3,3)\}$ on S. Then R is

(i) symmetric and but not anti-symmetric

(ii) anti-symmetric and but not symmetric

(iii) symmetric as well as anti-symmetric

(iv) neither symmetric nor anti-symmetric.

(b) Consider the poset $(\mathbb{N}, \leq)$. This poset has

(i) minimal element but no maximal element

(ii) maximal element but no minimal element

(iii) both minimal element and maximal element

(iv) neither minimal element nor maximal element.

(c) It is false that from a Hasse diagram of some poset

(i) minimal element(s) can be determined

(ii) maximal element(s) can be determined

(iii) both maximum and minimum element(s) can be determined

(iv) maximum and minimum element(s) can never be determined.

(d) Let $(B, \wedge, \vee)$ be a lattice and $a, b, c \in B$. Then dual of $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ is

(i) $a \wedge (b \vee c) = (a \wedge b) \wedge (a \wedge c)$ (ii) $a \wedge (b \wedge c) = (a \wedge b) \vee (a \wedge c)$

(iii) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ (iv) $a \vee (b \vee c) = (a \wedge b) \vee (a \vee c)$.

(e) A poset S is lattice if every pair of elements of its has

(i) greatest lower bound in S (ii) greatest lower bound and lowest upper bound in S

(iii) greatest and lowest element in S (iv) maximal and minimal elements in S.

(f) Let $\mathbb{N}$ be ordered by divisibility. Which subset of $\mathbb{N}$ is not linear (totally) order

(i) $\{24, 2, 4\}$ (ii) $\mathbb{N}$

(iii) $\{2, 8, 32, 4\}$ (iv) $\{7\}$.

(g) In the Boolean Algebra which one of the following is not true?

(i) $a + a = a$ (ii) $a + 1 = 0$

(iii) $a.(a + b) = a$ (iv) $(a + b)' = a'.b'$.

(h) The complement of $(x' + y)(x' + y')$ is

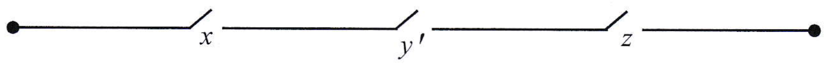
(i) $(x + y)(x + y')$ (ii) x

(iii) $(x' + y')y'$ (iv) None of these.

(i) Let $(B, +, \cdot, ')$ be a Boolean Algebra and $a, b \in B$. Then $a + (a' \cdot b) =$

(i) $a + b$ (ii) $a + b'$

(iii) $a' + b$ (iv) $a \cdot b$.

(j) 

Boolean expression corresponding to the above circuit is written as

(i) xyz (ii) $xy'z$

(iii) $x + y + z$ (iv) $x + y' + z$.

Group - B

(Marks : 60)

Answer ***any six*** questions.

2. (a) Define maximal and minimal element in a poset.

(b) Draw the Hasse diagram of the poset $(S, \leq)$, where $S = \{4, 12, 24, 48, 72\}$ and $a \leq b$ means a divides b for all $a, b \in S$. Find the greatest element (if exists) and least element (if exists) in $(S, \leq)$.

(c) Let $S = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$ and consider the poset $(S, \leq)$, where $a \leq b$ iff a divides b for all $a, b \in S$. Find the upper bound(s) and least upper bound of the set $\{6, 15\}$ in $(S, \leq)$. 2+(2+2)+(2+2)

3. (a) Define distributive lattice.

(b) Let S be the set of all positive divisors of 20 and a partial order relation $\leq$ is defined on S by $a \leq b$ iff a divides b. Examine whether $(S, \leq)$ is a distributive lattice.

(c) Let $(L, \wedge, \vee)$ be a distributive lattice and $a, b, c \in L$.

Prove that $a \wedge c = b \wedge c$ and $a \vee c = b \vee c \Rightarrow b = a$. 2+3+5

4. (a) Define Complete lattice. Prove that dual of a complete lattice is complete.

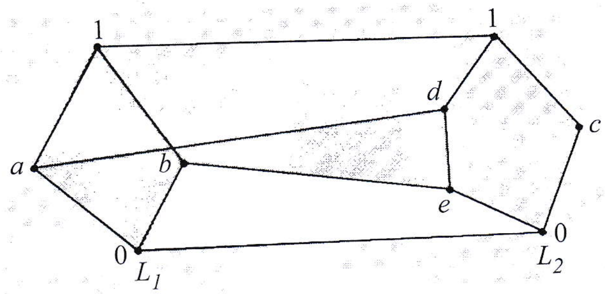
(b) Show that $(D_{30}, \leq)$ is a lattice. Find all sublattices of $(D_{30}, \leq)$.

(c) Prove that every finite lattice is bounded. (1+3)+(2+2)+2

5. (a) Prove that $\mathbb{N} \times \mathbb{N}$ is moduler lattice, where $\mathbb{N}$ is the chain of naturals under usual $\leq$.

(b) Define homomorphism between two lattices.

(c) Let us consider the mapping $\psi : (L_1, \wedge, \vee) \to (L_2, \wedge, \vee)$, where $L_1 = \{0, a, b, 1\}$ and $L_2 = \{0, c, d, e, 1\}$ be two lattices and ψ be defined as



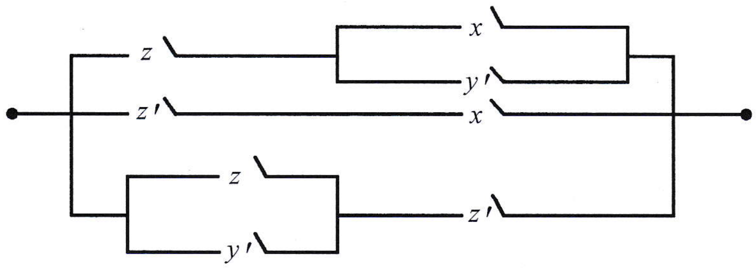
Show that ψ is homomorphism. 3+2+5

6. (a) From the following truth table write F in DNF and then simplify using Karnaugh Map :

x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

(b) Using Karnaugh Map find a minimal sum for $E = y't' + y'z't + x'y'zt + yzt'$. (2+3)+5

7. (a) Find a simpler equivalent circuit for the following :



(b) Find a switching circuit which realizes the switching function f given by the following switching table :

x	y	z	$f(x,y,z)$
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	1

5+5

8. (a) A committee consisting of three members approves any proposal by majority vote. Each member can approve a proposal by pressing a button attached to their seats. Design as simple a circuit as you can which will allow current to pass when and only when a proposal is approved.

(b) Construct a truth table for the Boolean expression : $xy' + y(x' + z) + z$. 5+5

9. (a) Prove that the following three expressions are equal :

(i) $(a + b)(a' + c)(b + c)$

(ii) $ac + a'b + bc$

(iii) $(a + b)(a' + c)$.

(b) What is Boolean polynomial? Give an example of Boolean polynomial. 6+(2+2)

10. (a) Let $(S, \leq)$ be a poset and $a, b \in S$. Prove that $a \vee b = b$ iff $a \wedge b = a$.

(b) Let $(L, \leq)$ be a lattice and $a, b \in L$. Show that $a \wedge (a \vee b) = a$.

(c) Give one example of a poset, where there are more than one minimal elements but no smallest element. 4+3+3