2022

STATISTICS — GENERAL

Paper : GE/CC-2

(Elementary Probability Theory)

(Group : 1 to 3)

Full Marks : 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer ***question nos.*** **1, 2** and ***any three*** questions from ***question nos.*** **3** to **7.**

1. Answer ***any five*** of the following : 2×5

(a) If a fair die is thrown once, construct two independent events A and B with non-zero probabilities.

(b) For any two events A and B, $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$. Find $P(A^c|B^c)$.

(c) A player rolls two die at a time. He wins Rs. 10 if the sum of the numbers obtained in a throw is eleven and loses Rs. 5 otherwise. Find his expected gain.

(d) For two independent events A and B if $P(A) = 0.7$ and $P(B) = 0.5$, find the probability of occurrence of exactly one of them.

(e) If a random variable X assumes only two values –1 and 1 such that $2P(X = 1) = P(X = -1)$, find $E(|X|)$.

(f) Find the mean of a geometric distribution based on number of trials with parameter $p = 0.3$.

(g) Let X be a random variable with pdf $f(x) = 3x^2$; $0 < x < 1$. Find the distribution function of X.

(h) If X follows $Bin\ (4, p)$ with $4P(X = 2) = P(X = 3)$, find $\text{Var}(X)$.

2. Answer ***any two*** of the following : 5×2

(a) Three identical boxes A, B and C contain respectively 2 white, 3 black balls; 4 white, 5 black balls and 3 white, 4 black balls. One ball is drawn at random from a box. If the ball is white, find the probability that it is drawn from box C.

(b) Let X be a continuous random variable with pdf $f(x)$ where

$$\begin{aligned} f(x) &= e^{-x},\ x > 0 \\ &= 0 \text{ otherwise,} \end{aligned}$$

Find the mean and median of X.

(c) Stating clearly the assumptions, show that Poisson distribution is a limiting case of binomial distribution.

Please Turn Over

3. (a) Give the statistical definition and the axiomatic definition of probability.

(b) What is the probability that four S's come consecutively when the letters of the word 'MISSISSIPPI' are arranged in all possible ways? (3+3)+4

4. (a) For three events A, B and C, find an expression for P(AUBUC).

(b) Find the mode of Bin(6, p). If $p = 0.3$, check whether the distribution is unimodal or not. 4+6

5. Show that for Poisson distribution with mean λ

$$\mu_{r+1} = \lambda\left(r\,\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right)$$

where μ_r denotes the r-th order central moment of this distribution. Hence find a measure of skewness and a measure of kurtosis of the distribution. 6+4

6. (a) Find the mean deviation about mean of a normal distribution.

(b) If μ_{2r} denotes the $2r$-th order central moment of a $N(\mu, \sigma^2)$ distribution, show that $\mu_{2r} = (2r\text{-}1)\mu_{2r-2}$. 4+6

7. (a) State and prove the weak law of large numbers (WLLN).

(b) Let X be a discrete random variable having probability distribution

x	:	0	1	2	3	4	5	6	7
$P(X = x)$	:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine the constant k. Also find $P(X > 5)$. 5+5