

2022

ECONOMICS — HONOURS

Paper : CC-4

Full Marks : 65

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

Group - A

1. Answer **any ten** questions :

2×10

- (a) Consider the function $f(x_1, x_2) = x_1x_2 + x_2^2$. Find the corresponding marginal functions and comment on their degree of homogeneity.
- (b) For the total cost function $C = y^2 + 10y + 25$; $y > 0$, show that when average cost (AC) curve is horizontal, then $AC = MC$ (Marginal Cost).
- (c) Find the point elasticity of demand (w.r.t. own price) for the demand function $x = 100p^{-2}$.
- (d) Find the extreme values of the function $y = 0.5x^3 - 3x^2 + 6x + 10$ and determine whether it gives a maxima or a minima.
- (e) Find the marginal product functions for the Cobb-Dauglas production function : $y = 10x_1^{1/2}x_2^{1/2}$.
- (f) For the function $f(x_1, x_2) = x_1^2x_2$, verify the Young's Theorem.
- (g) Determine the MRS for the utility function $u(x_1, x_2) = ax_1 + bx_2$.
- (h) Show that the quadratic equation formed by the following matrix product is positive definite.

$$[x_1 \ x_2] \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- (i) State the duality theorem in the context of linear programming problems.
- (j) Find the inflexion point for the function $y = \ln x + 1/x$.
- (k) A production function is given by : $Q(L) = 12L^2 - \frac{1}{20}L^3$; where L denotes the number of workers.

What size of workforce maximises output per worker?

- (l) For the function $x = 5.e^t$, show that the relative rate of increase $\frac{\dot{x}}{x}$ is constant.
- (m) Mohan lives in two periods, today and tomorrow. At the beginning of each period he earns ₹ 5,000. If the interest rate in each period is 0.25, find the present value of her lifetime income.

Please Turn Over

- (n) Write down the Kuhn-Tucker conditions for the following optimization problem :

$$\text{Maximize } z = 2x_1 - x_1^2 + x_2$$

$$\text{Subject to } 2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

- (o) Discuss the nature of the following time paths :

$$(i) y_t = 5 \left(-\frac{1}{10} \right)^t + 3$$

$$(ii) y_t = 2 \left(\frac{1}{3} \right)^t$$

Group - B

Answer **any three** questions.

2. Comment on the quasiconcavity/quasiconvexity of the following function :

$$y = 2x_1^{1/2} x_2^{1/2}; x_1, x_2 > 0.$$

5

3. Given $C = 102 + 0.7y$, $I = 150 - 100r$; $M_S = 300$; $M_T = 0.25y$, $M_P = 124 - 200r$ where C = consumption, Y = income, I = investment, r = rate of interest, M_S = Money supply, M_T = Transaction demand for money. M_P = Speculative demand for money. Find (i) The equilibrium level of income and the rate of interest. (ii) The levels of C , I , M_T and M_P at equilibrium.

3+2

4. What is a level curve? Compute the slope of the level curves for the function $f(x_1, x_2) = 2x_1 + 3x_2$.

2+3

5. Examine whether the following functions are homothetic?

$$(a) e^{x^2 y}$$

$$(b) 2 \log x + 3 \log y$$

2½+2½

6. A consumer's utility function is given by $U = x^\alpha y^\beta$. Show that the absolute value of price and income elasticities for either good is equal to unity.

5

Group - C

Answer **any three** questions.

7. (a) Derive the indirect utility function in case of a Cobb-Dauglas utility function $u(x, y) = x^\alpha y^\beta$. Where $\alpha + \beta = 1$ and the budget equation is given by : $I = P_x \cdot x + P_y \cdot y$.

- (b) Derive the compensated demand function for the utility function $u^o = q_1 q_2$ and the expenditure function $E = p_1 q_1 + p_2 q_2$. Verify the Shephard's Lemma.

4+(3+3)

8. (a) (i) Suppose that y is a function of x_1 and x_2 given by :

$$y = -(x_1 - 1)^2 - (x_2 - 2)^2 + 10$$

where y represents an individual's health (measured on a scale of 0 to 10), and x_1 and x_2 might be daily dosage of two health enhancing drugs. The objective is to maximise y . But the choice of x_1 and x_2 is constrained by the fact that an individual can tolerate only one drug does per day i.e $x_1 + x_2 = 1$. Find out the optimal combination of drug that will maximise the health standard subject to the constraint.

- (ii) What would have been the optimal choice had there been no constraint. How does the maximum value of y changes in this unconstrained case, compared to the constrained one.

- (b) Consider the following profit equation of a firm producing two products x and y :

$$\pi = 80x - 1.5x^2 - xy - y^2 + 60y$$

Find the profit maximizing combination of output and the level of maximum profit of the firm.

(2+3)+5

9. (a) Let the demand and supply function for a commodity be :

$$Q_d = D(P, t_0); \frac{\partial D}{\partial P} < 0, \frac{\partial D}{\partial t_0} > 0 \text{ and } Q_s = Q_{s_0}. \text{ Where } t_0 \text{ is consumer's taste for the commodity and}$$

where both partial derivatives are continuous.

- (i) Write the equilibrium condition as a single equation.
 (ii) Is the implicit function applicable?
 (iii) How would the equilibrium price vary with consumer taste.
- (b) A firm uses capital K , labour L and land T to produce Q units of output, where $Q = K^{2/3} + L^{1/2} + T^{1/3}$. Suppose that the firm is paid a positive price p for each unit it produces and the positive prices it pays per unit of capital, labour and land are r , w and q respectively.
- (i) Find the values of K , L and T that maximises firm's profit.
 (ii) Show that $\frac{\partial Q^*}{\partial r} = -\frac{\partial K^*}{\partial p}$, where Q^* denotes the optimal level of output and K^* denotes the optimal level of capital stock.

(1+2+2)+(3+2)

10. (a) Consider the following market model :

$$Q_t^d = Q_t^s \quad Q_t^d = 20 - 3P_t$$

$$Q_t^s = \begin{cases} -10 + 3P_t^* & \text{for } t = 1, 2, \dots \\ 0 & \text{for } t = 0 \end{cases}$$

where P_t^* is the expected price at t -th period given that

$$P_t^* = P_{t-1}^* + \alpha [P_{t-1} - P_{t-1}^*] \quad 0 < \alpha < 1 \quad P_t^* = P_0; t = 1$$

Find the time path of price.

Please Turn Over

(b) Consider the linear difference equation of the Cob-Web model :

$$P_{t+1} = \frac{a+\gamma}{\beta} - \frac{\partial}{\beta} \cdot P_t \left(\frac{\partial}{\beta} > 0 \right)$$

Draw a phase line to ascertain the nature of the time path.

5+5

11. Consider the following linear programming problem :

$$\text{Maximize } x_1 + x_2$$

$$\text{Subject to } x_1 + 2x_2 \leq 14$$

$$2x_1 + x_2 \leq 13$$

$$x_1 \geq 0, x_2 \geq 0$$

(a) Solve the problem graphically.

(b) Write down the dual of this problem.

(c) Use complementary slackness conditions to solve the dual.

(d) Check whether duality theorem holds.

(3+2+3+2)