2021

ECONOMICS — HONOURS

Paper : CC-2

(Mathematical Methods for Economics - I)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Section – A

1. Answer ***any ten*** questions : 2×10

(a) A and B are two sets and A×B consists of six elements. If three of the elements of A×B are (2, 5), (3, 7), (4, 7), find all the elements of A×B.

(b) Solve the following simultaneous equations for x and y :

$$px + (1 - q)\, y = R$$

$$qx + (1 - p)\, y = S$$

(c) A 5 kg. iron ball is to be produced. It is necessary that the weight of the ball does not deviate more than 1 gm from its stated weight. Write a specification for the ball's weight in kg by (i) using inequality signs, (ii) with the aid of an absolute value sign.

(d) Consider an open cardboard box whose dimensions are, length = $(18 - 2x)$cm, breadth = $(18 - 2x)$ cm and height = x cm. What should be the value of x (in cm) so that the volume of the box is maximised?

(e) A survey revealed that 50 semester-1 students liked studying Microeconomics, 40 liked studying Mathematical Methods, 35 liked studying both and 10 did not like studying either. How many students took the survey?

(f) Only for special additive functions it is true that $f(a + b) = f(a) + f(b)$ for all 'a' and 'b'. Determine whether the following are special additive functions : (i) $f(x) = 2x^2$, (ii) $f(x) = \sqrt{x}$.

(g) Find the domain and range of $g(x) = 1 - \sqrt{x + 2}$.

(h) Let $f(x) = \dfrac{ax + b}{cx - a}$, where a, b and c are constants and $c \neq 0$. Assuming that $x \neq \left(a/c\right)$, find $f\left(\dfrac{ax + b}{cx - a}\right)$.

(i) Find $\lim_{x \to 1} \frac{x^n - 1}{x - 1}$ (n is a natural number).

(j) If $f(x) = \frac{2x}{x^2 + 2}$, for what value of x is $f'(x) = 0$?

(k) Find $\lim_{x \to 0+} \frac{x + |x|}{x}$.

(l) For what value of 'a' is the following function continuous everywhere?

$$f(x) = \begin{cases} ax^2 + 4x - 1, \text{ if } x \leq 1 \\ -x + 3 \quad , \text{ if } x > 1 \end{cases}$$

(m) Check whether $f(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$ is a concave or convex function.

(n) If $c(x)$ is the cost function indicating the cost incurved for producing x units of a commodity. If marginal cost $c'(x) = 3x + 4$ and fixed cost, $c(0) = 40$. Find $c(x)$.

(o) What is a saddle point in a mathematical game?

Section – B

Answer ***any three*** questions.

2. Assuming that each of the following functions are linear, give an economic interpretation of the slope of the following functions : 2+3

(a) $C(Y)$ is the total national consumption when national income is Y;

(b) $F(Q)$ is the local cost for producing Q units of output.

3. An employee gets a salary according to a contract that establishes a relationship between pay and the levels of sales made by the employee. The contract stipulates that the salary will be composed of three parts :

(a) ₹ 500 as basic amount,

(b) Commission of 10% over sales,

(c) ₹ 200 bonus if the sales exceed ₹ 10,000.

From the function linking salary to sales, draw the graph of the function. 1+2+2

4. Given the demand function $P = 8.25e^{-0.02Q}$. Determine the quantity and price at which TR will be maximized. 2½+2½

5. Find the saddle point for the following game : 5

Strategies ↘	*Player B*	
Player A	To be attentive	Not to be attentive
To teach well	4, 4	3, –1
Not to teach well	1, 2	2, 1

6. The Total cost (C) of a firm is given by :

$$C = 100q - 80q^2 + \frac{1}{3}q^3$$

where q is the quantity produced.

(a) Find the *MC* of production.

(b) Find the slope of the *MC* function.

(c) At what value of q does *MC* equal *AC* ? 1+2+2

Section – C

Answer ***any three*** questions.

7. (a) Check for quasi-convexity and quasi-concavity for the given function :

$$y = x^2 \ (x \geq 0)$$

(b) Examine the continuity and differentiability of $f(x) = 2x + |x-1|$, at $x = 1$. 5+5

8. (a) By applying maximin-minimax criteria, find the saddle point if any of the following game whose pay-off matrix for player A is given below :

Strategies ↘	*Player B*			
Player A	B_1	B_2	B_3	B_4
A_1	9	2	19	5
A_2	6	5	7	16
A_3	7	3	4	10

(b) Find the inverse for

$$A = \begin{bmatrix} 4 & 1 & -5 \\ -2 & 3 & 1 \\ 3 & -1 & 4 \end{bmatrix}$$

5+5

9. Given the input-coefficient matrix of an input-output model and the final consumption demand vector :

$$A = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.2 \end{bmatrix}, \; C = \begin{bmatrix} 50 \\ 50 \end{bmatrix}$$

Find the optimum output levels of the two goods. Suppose that 0.2 and 0.1 are labour coefficients of the two goods respectively. If $\bar{L} = 100$, will there be unemployment in the economy? 5+5

10. (a) A survey of 500 students taking one or more courses in Chemistry, Physics and Statistics during one semester revealed the following number of students in the indicated subjects :

Chemistry 329, Chemistry and Physics 83, Physics 189, Chemistry and Statistics 217, Statistics 295, Physics and Statistics 63.

How many students were taking (i) All three subjects (ii) Chemistry but not Statistics (iii) Chemistry but not Physics or Statistics?

(b) Use the concept of dominance and solve the following game whose pay-off matrix for player A is given below :

Strategies ↘	*Player B*		
Player A	B_1	B_2	B_3
A_1	4	3	6
A_2	6	5	8
A_3	7	1	5

5+5

11. (a) Find $\frac{dy}{dx}$ for the following equation :

$$4x^2 - y^3 = 97.$$

(b) Consider the following game :

Strategies ↘	*Player B*		
Player A	B_1	B_2	B_3
A_1	1, 0	1, 3	3, 0
A_2	0, 2	0, 1	3, 0
A_3	0, 2	2, 4	5, 4

Is there any dominant strategy for each player? Is there any pure strategy Nash equilibrium?

3+(3+4)