2021

STATISTICS — GENERAL

Paper : GE/CC-2

(Elementary Probability Theory)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

All notations and symbols have their usual meanings.

Answer ***question nos.*** **1**, **2**; and ***any three*** questions from ***question nos.*** **3** to **7**.

1. Answer ***any five*** of the following : 2×5

(a) If the events A, B and C are exhaustive, find the probabilities that (i) at least one of them occurs and (ii) none of them occurs.

(b) If 3 unbiased coins are tossed simultaneously, describe the sample space and find the probability that at most one head occurs.

(c) For two events A and B with $P(B) > 0$, show that $P(A|B) \geq \dfrac{P(A)+P(B)-1}{P(B)}$.

(d) If X is a symmetric binomial variable with $n = 12$, calculate $E[X(X-1)]$.

(e) If A and B are independent events, show that the events A^c and B^c are also independent.

(f) What do you mean by Bernoulli trials?

(g) The probability density function of a random variable X is $f(x) = 2e^{-2x}$, $x > 0$. Find $E(X)$.

(h) If a Poisson random variable X has two modes at $x = 2$ and $x = 3$, find the coefficient of variation of X.

2. Answer ***any two*** of the following : 5×2

(a) In a sample space with four equally likely sample points, define 3 events A, B and C so that they are pairwise independent but not mutually independent.

(b) Find the points of inflection of a normal distribution having mean μ and variance σ^2.

(c) A continuous random variable X has probability density function :

$$f(x) = \begin{cases} kx^3(4-x^2), & 0 \leq x \leq 4, k > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Find k and $\text{Var}(X)$.

3. (a) Give the classical definition of probability. What are its limitations?

(b) If A and B are independent events with $P(A) > \frac{1}{2}, P(B) > \frac{1}{2}, P(A \cap B^c) = \frac{3}{25}$ and $P(A^c \cap B) = \frac{8}{25}$, find the value of $P(A)$ and $P(B)$. (2+3)+5

4. (a) A candidate is interviewed for three posts. For the first post there are 3 candidates, for the second there are 4 and for the third there are 2. What is the probability of his getting at least one post?

(b) State and prove Bayes' Theorem in probability theory. 4+6

5. (a) Define a random variable with an example. When will it be discrete or continuous?

(b) Find the mean and variance of X with probability density function

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise.} \end{cases}$$

6+4

6. (a) For a binomial distribution with mean np $(0 < p < 1)$ and k-th order central moment μ_k, show that,

$$\mu_{k+1} = p(1-p)\left[nk\mu_{k-1} + \frac{d\mu_k}{dp}\right].$$

(b) If X is a Poisson random variable with parameter μ such that $P(X = 2) = 2P(X = 3)$, find $P(X > 0 | X \le 2)$ and $P(X = \text{at most } 1)$. 6+4

7. (a) State and prove Chebyshev's inequality.

(b) Find a lower bound of $P\left[\left|X - \frac{1}{2}\right| \le \frac{1}{2}\right]$, where X follows uniform (0, 1) distribution. 6+4

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