2023

MATHEMATICS — GENERAL

Paper : GE/CC-4

Full Marks : 65

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

The symbols and notations have their usual meanings.

Group - A

1. Choose the correct alternative : 1×10

   (a) For what real values of 'α' does the set $S = \{(\alpha, 0, 1), (1, \alpha + 1, 1), (1, 1, 1)\}$ of $\mathbb{R}^3$ is linearly independent?

   (i) $\mathbb{R} - \{-1, 1\}$ (ii) $\mathbb{R} - \{-1, 0\}$

   (iii) $\mathbb{R} - \{0, 1\}$ (iv) $\mathbb{R} - \{-1, 0, 1\}$.

   (b) In a Ring $(R, \circ, *)$, where $x \circ y = x + y - 1$, $x * y = x + y - xy$, $\forall x, y \in R$, the zero element is

   (i) -1 (ii) 0

   (iii) 2 (iv) 1.

   (c) The eigenvalues of the square matrix $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 5 \end{pmatrix}$ are

   (i) all real (ii) all purely imaginary

   (iii) all purely imaginary or zero (iv) either purely real or purely imaginary.

   (d) If for two events A and B, $P(A) = \frac{1}{2}$; $P(B) = \frac{2}{5}$; $P(AB) = \frac{3}{10}$, then $P(\bar{A} \mid B)$ is

   (i) $\frac{1}{3}$ (ii) $\frac{2}{3}$

   (iii) $\frac{1}{2}$ (iv) $\frac{1}{4}$.

Please Turn Over

(e) For what value of 'β' the function $f(x)$ defined by

$$f(x) = \begin{cases} \beta x(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function?

   (i) 6   (ii) 5

   (iii) 2   (iv) 4.

(f) If X and Y are two random variables such that $\operatorname{Var} X = 16$, $\operatorname{Var} Y = 9$ and the correlation coefficient is $\frac{3}{4}$, then value of $\operatorname{Cov}(X, Y)$ is

   (i) 9   (ii) 16

   (iii) 12   (iv) 24.

(g) A random variable X has probability density function

$$f(x) = \begin{cases} \dfrac{1}{6}, & -3 < x < 3 \\ 0, & \text{elsewhere} \end{cases}$$

Then $P(X > 1)$ is given by

   (i) $\dfrac{1}{3}$ (ii) $\dfrac{1}{2}$

   (iii) $\dfrac{1}{6}$ (iv) $\dfrac{3}{4}$.

(h) The value of the FORTRAN expression : $J + L * I/3 + J/I - K * * L/I$, where $I = 4$, $J = 5$, $K = 3$ and $L = 2$ is

   (i) 5   (ii) 6

   (iii) 7   (iv) -6.

(i) FORTRAN expression of $\log_e \sqrt{\dfrac{a}{bc}} + \log_{10} |a|$ is

   (i) ALOG(SQRT($A/(B * C)$)) + ALOG10(ABS(A))

   (ii) ALOG10(SQRT($A/(B * C)$)) + ALOG(ABS(A))

   (iii) LOG(SQRT($A/(B * C)$)) + LOG10(ABS(A))

   (iv) LOGE (SQRT($A/(B * C)$)) + ALOG10(ABS(A)).

(j) Octal number corresponding to the binary number $(1101.10111)_2$ is

   (i) $(15.52)_8$ (ii) $(14.56)_8$

   (iii) $(15.56)_8$ (iv) $(15.46)_8$.

Group - B

Unit - 1

(Algebra - II)

2. Answer *any three* questions :

(a) Let $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \text{ and } ad - bc = 1 \right\}$. Show that M forms a group under matrix

multiplication.   5

(b) Define a subring of a ring $(R, +, \cdot)$. Check whether $S = \{2n : n \in \mathbb{Z}\}$ is a subring of the ring
   $(\mathbb{Z}, +, \cdot)$ of integers.   5

(c) Let $W = \{(x, y, z) \in \mathbb{R}^3 : x - 4y + 3z = 0\}$. Show that W is a subspace of $\mathbb{R}^3$. Find the basis and
   dimension of W.   3+1+1

(d) Examine whether the real quadratic form $5x^2 + y^2 + 3z^2 + 4xy - 2yz - 2zx$ is positive definite or
   not.   5

(e) Verify Cayley-Hamilton's theorem for the following matrix A given by $A = \begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$. If possible

   find A^{-1} from the result.   3+2

Unit - 2

(Computer Science and Programming)

3. Answer *any four* questions :

(a) Evaluate the octal arithmetic $(576)_8 + (116)_8 - (477)_8$.   5

(b) Write an algorithm to find HCF and LCM of two distinct positive integers 'm' and 'n'.   5

(c) Write a flowchart for finding the maximum of n given real numbers.   5

(d) (i) Write the full form of the terms of BIT and ASCII.

   (ii) Write three differences between high level language and assembly language.   2+3

(e) (i) Write the FORTRAN expression of $\sqrt{x^3 + y^2} \tan(y/x) + \dfrac{7}{3\sqrt{2}} \log_e z$.

   (ii) Let $A = 4.7$, $B = 5.6$ and $M = ABS(A - 4.0 * B)/7$. Find what value of M will be stored.   3+2

(f) Write a FORTRAN program to find the sum of squares of first 10 natural numbers.   5

(g) Write a FORTRAN program to find the area, perimeter and diagonal of a rectangle whose
   two adjacent sides are x and y.   5

Please Turn Over

Unit - 3

(Probability and Statistics)

4. Answer *any four* questions :

(a) There are two urns *A* and *B*. The urn *A* contains 3 white and 4 red balls while the urn *B* contains 4 white and 3 red balls. One ball is transferred from urn *A* to urn *B* and a ball is drawn from urn *B*. What is the probability that the ball is red? 5

(b) Find the mean and variance of a Binomial distribution with parameters *n* and *p*. 5

(c) There are 4 red and 6 blue balls in an urn. A man draws 2 balls at a time at random. He will get ₹ 15.00 if the balls are of same colour and he will pay ₹ 10.00 if the balls are of different colours. Find the expected value of the money which the man will receive. 5

(d) Construct a frequency distribution table with class intervals $50 - 69$, $70 - 89$, $90 - 109$, $110 - 119$, ... from the following data :

95, 131, 53, 117, 155, 100, 65, 122, 71, 78, 153, 90, 125, 80, 105, 137, 65, 136, 145, 120.

Also draw the histogram for the above distribution in plane paper. 3+2

(e) If two regression lines involving two variables *x* and *y* are $y = 5.6 + 1.2x$ and $x = 12.5 + 0.6y$. Find the mean of *x*, *y* and their correlation coefficient. 5

(f) Define an unbiased and consistent estimate of a parameter in population distribution. For a random sample of size *n* for any (m, σ) population, prove that the sample mean is an unbiased estimate of population mean. 1+1+3

(g) Find a 95% confidence interval for the mean of a normal distribution with $\sigma = 3$, given the sample $(2.3, -0.2, -0.4, -0.9)$. Given $P(U \geq 1.96) = 0.025$, where *U* is $N(0, 1)$ variate. 5

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