2023

STATISTICS — GENERAL

Paper : GE/CC-2

(Elementary Probability Theory)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

All notations and symbols have their usual meaning.

Answer ***question nos.*** **1**, **2** and ***any three*** questions from ***question nos.*** **3** to **7**.

1. Answer ***any five*** of the following : 2×5

(a) State two limitations of the classical definition of probability.

(b) X has pdf $f(x)=4x^3$; $0<x<1$. Find the cdf of X.

(c) If $P(A_1)=\frac{1}{2}$, $P(A_2)=\frac{3}{5}$ and $P(A_1 \cap A_2)=\frac{1}{3}$, then find $P(A_1^c \cap A_2^c)$.

(d) The mean of a symmetric binomial distribution is 5. What is its variance?

(e) A Poisson distribution has double mode at $X=2$ and $X=3$. What is the coefficient of variation of the distribution?

(f) $X \sim R(\alpha, \beta)$. Find $E(X)$.

(g) Write the points of inflection for N(2, 8) distribution.

(h) In how many different ways 10 apples can be divided among 5 people?

2. Answer ***any two*** of the following : 5×2

(a) Write a short note on statistical definition of probability.

(b) Find the mean and variance for Poisson (2λ) distribution.

(c) Show that for the exponential distribution defined by the pdf

$$f(x)=\begin{cases}\theta e^{-\theta x}, & x \geq 0 \\ 0, & \text{otherwise}\end{cases} \quad ; \theta > 0,$$

$P(X > m+n \mid X > n) = P(X > m)$.

3. (a) Define Mutually Exclusive and Independent Events with example.

(b) There are two events A and B such that $P(A) > 0$ and $P(B) > 0$. Prove that A and B cannot be independent if they are mutually exclusive.

(c) Find the mean and variance of a discrete uniform distribution. 4+2+4

4. (a) Let $X \sim$ Binomial (n, p). Prove that $\mu_{r+1} = pq\left[r\mu_{r-1} + \frac{d}{dp}\mu_r\right]$, where μ_r is the r^{th} order central moment.

(b) (i) Write Axiomatic definition of probability.

(ii) Prove $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ by using the axioms only. 6+(2+2)

5. (a) State and prove Bayes' Theorem.

(b) The chance that doctor A will diagnose a disease X correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X, died. What is the chance that his disease was diagnosed correctly? 5+5

6. (a) Define discrete and continuous random variables with example.

(b) Find the mode and one measure of skewness of a normal distribution with mean 'μ' and variance 'σ^2'. 4+(4+2)

7. The pmf of a random variable X is

$$f(x) = \begin{cases} k, & \text{for } x = 0 \\ 2k - 3k^2, & \text{for } x = 1 \\ 4k - 1, & \text{for } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) Determine the value of k.

(b) Find $P(X > 0 \mid X < 2)$.

(c) Find expectation and variance of X. 4+2+(2+2)
