MATH-H-CC 3-3-TH
Real Analysis

Full Marks: 100 (Theory: 75 and Tutorial:25)

Group A
[Marks: 30][24 classes]

• Intuitive idea of real numbers. Mathematical operations and usual order of real numbers revisited with their properties (closure, commutative, associative, identity, inverse, distributive). Idea of countable sets, uncountable sets and uncountability of $\mathbb{R}$. Concept of bounded and unbounded sets in $\mathbb{R}$. L.U.B. (supremum), G.L.B. (infimum) of a set and their properties. L.U.B. axiom or order completeness axiom. Archimedean property of $\mathbb{R}$. Density of rational (and Irrational) numbers in $\mathbb{R}$.
• Intervals. Neighbourhood of a point. Interior point. Open set. Union, intersection of open sets. Limit point and isolated point of a set. Bolzano-Weierstrass theorem for sets. Existence of limit point of every uncountable set as a consequence of Bolzano-Weierstrass theorem. Derived set. Closed set (defined as Complement of open set). Union and intersection of closed sets as a consequence. No nonempty proper subset of $\mathbb{R}$ is both open and closed. Expressing an open set of $\mathbb{R}$ as countable union of disjoint open intervals (statement only). Dense set in $\mathbb{R}$ as a set having non-empty intersection with every open interval. $\mathbb{Q}$ and $\mathbb{R}\backslash\mathbb{Q}$ are dense in $\mathbb{R}$.

Group B
[Marks: 35][28 classes]

• Real sequence. Bounded sequence. Convergence and non-convergence. Examples. Boundedness of convergent sequence. Uniqueness of limit. Algebra of limits.
• Relation between the limit point of a set and the limit of a convergent sequence of distinct elements. Monotone sequences and their convergence. Sandwich rule. Nested interval theorem. Limit of some important sequences : $\left\{n^{\frac{1}{n}}\right\}_n$, $\{x^n\}_n$, $\left\{x^{1/n}\right\}_n$, $\{x_n\}_n$ with $\frac{x_{n+1}}{x_n} \to l$ and $|l| < 1$, $\left\{\left(1+\frac{1}{n}\right)^n\right\}_n$, $\left\{1+\frac{1}{1!}+\frac{1}{2!}+\cdots+\frac{1}{n!}\right\}_n$, $\{a^{x_n}\}_n$ $(a > 0)$. Cauchy's first and second limit theorems.
• Subsequence. Subsequential limits, lim sup as the L.U.B. and lim inf as the G.L.B of a set containing all the subsequential limits. Alternative definition of lim sup and lim inf of a sequence using inequality or as $lim\,sup\,x_n$ =

$\inf_n sup\{x_n, x_{n+1}, \ldots,\}$ and $lim\ inf\ x_n = \sup_n inf\{x_n, x_{n+1}, \ldots,\}$ [Equivalence between these definitions is assumed]. A bounded sequence $\{x_n\}_n$ is convergent if and only if $lim\ sup\ x_n = lim\ inf\ x_n$. Every sequence has a monotone subsequence. Bolzano-Weierstrass theorem for sequence. Cauchy sequence. Cauchy's general principle of Convergence.

Group C
[Marks: 10][8 classes]

• Infinite series, convergence and non-convergence of infinite series, Cauchy criterion, tests for convergence; comparison test, limit comparison test, ratio test, Cauchy's nth root test, Kummer's test (statement and problems), Raabe's test (statement and problems), Gauss test (statement and problems). Alternating series, Leibniz test. Absolute and conditional convergence, Riemann's rearrangement theorem (statement and problems).

References

[1] R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, 3rd Ed., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.
[2] G. G. Bilodeau, P. R. Thie, G.E. Keough, An Introduction to Analysis, 2nd Ed., Jones & Bartlett, 2010.
[3] B. S. Thomson, A. M. Bruckner and J. B. Bruckner, Elementary Real Analysis, Prentice Hall, 2001.
[4] S. K. Berberian, A First Course in Real Analysis, Springer Verlag, New York, 1994.
[5] T. M. Apostol, Mathematical Analysis, Narosa Publishing House, 2002.
[6] R. Courant and F. John, Introduction to Calculus and Analysis, Vol I, Interscience Publishers, 1965.
[7] W. Rudin, Principles of Mathematical Analysis, Tata McGraw-Hill, 1976.
[8] C. C. Pugh, Real Mathematical Analysis, Springer, 2002.
[9] T. Tao, Analysis I, Hindustan Book Agency, 2006.
[10] R. R. Goldberg, Methods of Real Analysis, John Wiley & Sons, 1976.
[11] H. R. Beyer, Calculus and Analysis, Wiley, 2010.

MATH-H-CC 4-3-TH
Ordinary Differential Equations – I
and Group Theory - I

Full Marks: 100 (Theory: 75 and Tutorial: 25)

Group A: Ordinary Differential Equations – I

[Marks: 45][36 classes]

- Formation of differential equations, order and degree of a differential equation, First order and first degree differential equations; Homogeneous and exact differential equations, conditions for an equation of the first order to be exact, Integrating factors, Rules for finding integrating factors, Linear equations and Bernoulli equations.

- First order higher degree differential equations solvable for x, y and p, Clairaut's forms. Singular solutions, Equations of tac-locus, nodal locus, cuspidal locus.

- Higher order linear and nonlinear equations, Concept of Wronskian and its properties, Complementary functions, Particular integrals, linear homogeneous and non-homogeneous equations with constant coefficients, Method of undetermined coefficients, Method of variation of parameters. Simultaneous linear differential equations.

- Higher order linear equations with variable coefficients reducible to linear equations with constant coefficients (Euler's equation), Condition for exactness of higher order linear equations, Integrating factors, Equations of the form $\frac{d^n y}{dx^n} = f(y)$ $(n \geq 2)$.

Group-B: Group Theory – I
[Marks: 30][24 classes]

- Definition of a group, examples of groups including permutation groups, dihedral groups and quaternion groups (through matrices), elementary properties of groups, examples of commutative and non-commutative groups. Subgroups and examples of subgroups, necessary and sufficient condition for a nonempty subset of a group to be a subgroup, Normalizer, centralizer, center of a group, product of subgroups.

- Order of an element of a group, order of a group, cyclic group, properties of cyclic groups, classification of subgroups of cyclic groups, Permutation, cycle notation for permutations, properties of permutation, even and odd permutations, Alternating group, properties of cosets, Lagrange's theorem and consequences including Fermat's little theorem.