2022

MATHEMATICS — GENERAL

Paper : GE/CC-4

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Choose the correct answer : 1×10

(a) Which of the following set is a group with respect to addition

(i) $\{-3, -2, -1, 0, 1, 2, 3\}$ (ii) $\{-1, 1\}$

(iii) $\{-1, 0, 1\}$ (iv) $\{0\}$.

(b) -2 is an eigenvalue of the matrix $M = \begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$. Then M^{-1} has an eigenvalue

(i) -2 (ii) 1

(iii) 2 (iv) $-\frac{1}{2}$

(c) Probability that **at least one** of the events A and B occurs is

(i) $P(A) + P(B) - P(AB)$ (ii) $P(A) + P(B) + 2P(AB)$

(iii) $P(A) + P(B) + P(AB)$ (iv) $P(A) + P(B) - 2P(AB)$

(d) Number of divisor of zero in the Ring $(\mathbb{Z}_5, \oplus, \odot)$ is

(i) 0 (ii) 1

(iii) 2 (iv) 3

(e) If $(0, 1, 3) = a(2, 1, 1) + b(4, 2, 2)$, then the values of a and b are

(i) $(1, 1)$ (ii) $(-1, 1)$

(iii) $(0, 0)$ (iv) None of these.

(f) For the probability density function given by $f(x)=\begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

the mean is

(i) 1 (ii) $\frac{1}{2}$

(iii) 2 (iv) 4

(g) If $E(T_1)=\theta_1+\theta_2$ and $E(T_2)=\theta_1-\theta_2$, then the unbiased estimator of θ_1 is

(i) T_1+T_2 (ii) $\frac{1}{2}(T_1-T_2)$

(iii) $\frac{1}{2}(T_1+T_2)$ (iv) $\frac{1}{2}(T_2-T_1)$

(h) Binary number corresponding to the decimal number 27.625 is

(i) 11011.101 (ii) 10111.101

(iii) 11101.011 (iv) 11011.011

(i) Which of the following can be a variable name in C?

(i) Volatile (ii) True

(iii) Friend (iv) Export.

(j) The value of the FORTRAN expression : $(A*(B+C))/D+A$, where $A=3$, $B=5$, $C=-2$ and $D=4$ is

(i) 3 (ii) 4

(iii) 5 (iv) 6

Group-B

Unit-1

(Algebra - II)

2. Answer ***any three*** questions : 5×3

(a) Prove that the set $Q\setminus\{-1\}$ is a group with respect to the composition 'o' defined by $aob=a+b+ab$. Is it abelian?

(b) Show that the ring of matrix $\left[\begin{pmatrix} a & b \\ 2b & a \end{pmatrix} : a,b\in\mathbb{R}\right]$ does not form a Field, $\mathbb{R}$ being the set of all real numbers.

(c) Is the set $U=\{(x,y,z)\in\mathbb{R}^3 : x-2y+3z=0\}$ a subspace of the real vector space $\mathbb{R}^3$? If so, find the basis and dimension of this subspace.

(d) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}$.

(e) Show that the real quadratic form $5x^2 + y^2 + 14z^2 - 4yz - 10zx$ is positive definite.

Unit-2

(Computer Science and Programming)

3. Answer ***any four*** questions :

(a) Find the product of $(11.0011)_2$ and $(10.01)_2$ and also find the octal and hexadecimal equivalents of the product. 5

(b) Draw a flowchart for computing the g.c.d. of two positive integers m and n. 5

(c) (i) Let A = 2.7, B = 3.5 and L = ABS (A – 3.*B)/5. Find what will be stored at L.

(ii) Write FORTRAN expression of $\dfrac{\sqrt{a+\log_e b}}{c+d\sin x}$ 2+3

(d) Write an algorithm to sort n given integers in descending order. 5

(e) Write a FORTRAN program to find the area of a triangle whose three sides are given. 5

(f) What is positional number system? Why are binary numbers used in computer design? 2+3

(g) Write a FORTRAN program to check whether a year is a Leap year or not. 5

Unit-3

(Probability and Statistics)

4. Answer ***any four*** questions : 5×4

(a) Bag A contains 2 white and 3 red balls; and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag B.

(b) Four persons are chosen at random from a group containing 3 men, 2 women and 4 children. Show that the chances that exactly two of them will be children is $\frac{10}{21}$.

(c) Find the coefficient of correlation from the following data :

x	0	1	2	3	4
f	2	3	5	10	5

(d) Draw a Histogram from the following distribution :

Age Group	14 – 15	16 – 17	18 – 20	21 – 24	25 – 29	30 – 34	35 – 39
No. of wage earners	60	140	150	110	110	100	90

(e) The population of scores of 10 years children in a test is known to have a standard deviation 5.2. If a random sample of size 20 shows a mean of 16.9, find 95% confidence interval for the mean score of the population, assuming that the population is normal.

$$\left(\text{Given that } \frac{1}{\sqrt{2\pi}} \int_{1.96}^{\infty} e^{-\frac{x^2}{2}} dx = 0.025 \right).$$

(f) If the equations of two regression lines obtained in a correlation analysis are $2y + x = 11$ and $2x + 3y - 18 = 0$, determine which one of them is the regression equation of x on y. Find the means and correlation coefficient of x and y.

(g) In a random sample of size 400 there are 80 defective items. Test at 5% level whether the proportion of defective items in the population may be regarded as $\frac{1}{6}$.

$$\left[\text{Given } \int_{0}^{1.96} \phi(t)dt = 0.475, \phi \text{ is the pdf of normal variate} \right].$$